Phase Collapse in Neural Networks

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John Zarka

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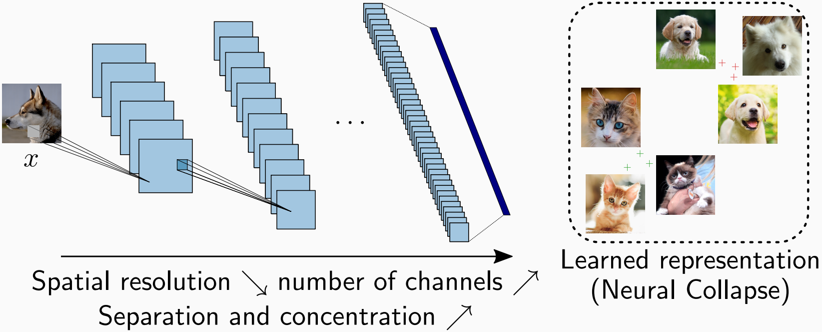
Neural collapse

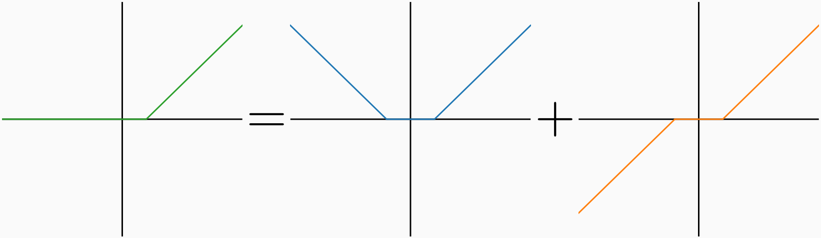


CNN classifiers simultaneously move spatial information into channels and increase linear separation

Can we define a non-linear operator with these properties?

Decomposition of ReLU

ReLUs can can be separated in two opposite non-linearities with an even-odd decomposition:



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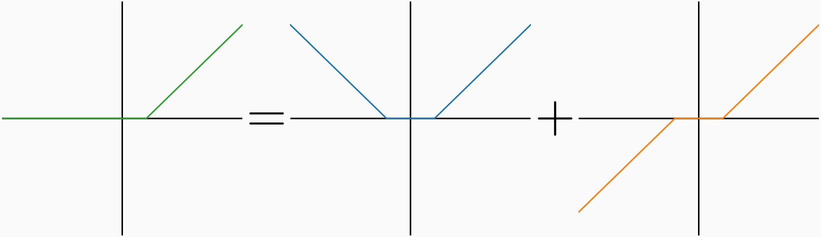

- Absolute value: collapses the sign, preserves the amplitude

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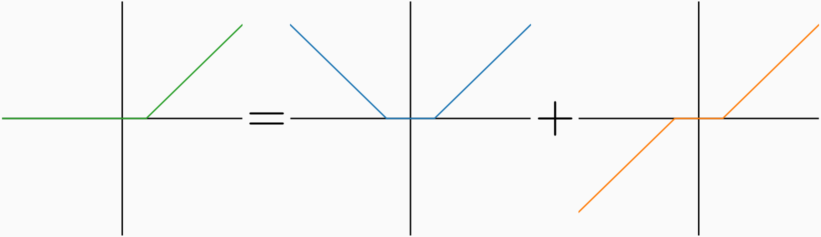
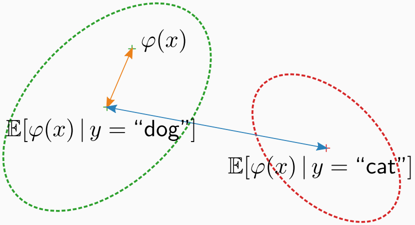


- Absolute value: collapses the sign, preserves the amplitude
- Soft-thresholding: preserves the sign, thresholds the amplitude

Concentration with soft-thresholding



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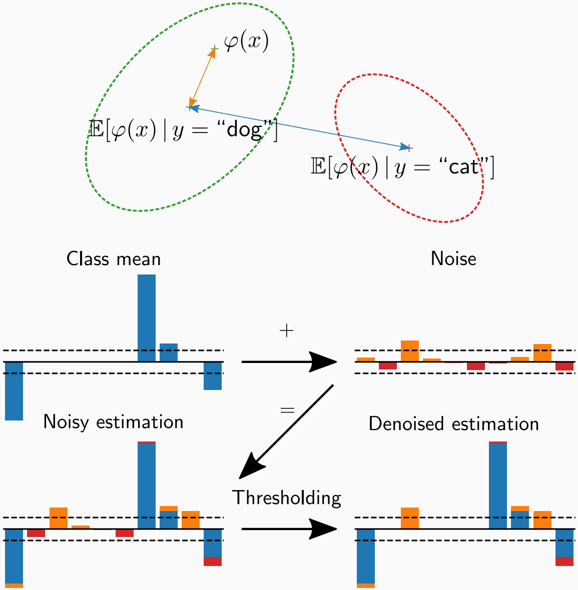

Separation with phase collapse

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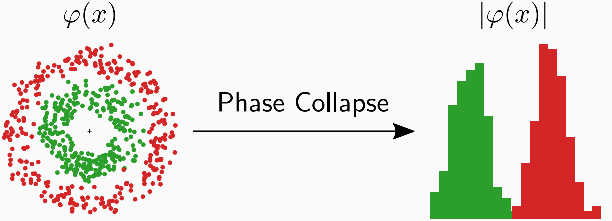
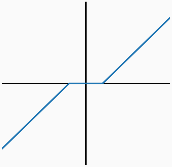
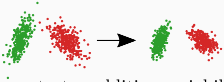


Comparison between sparsity and phase collapse

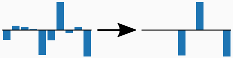
Concentration with soft-thresholding



Odd part of ReLU
Collapses small amplitudes

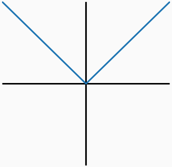


Concentrates additive variability
Does not separate class means

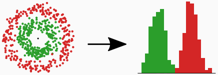


Performs denoising
Cannot be further sparsified

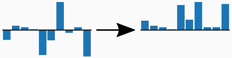
Separation with complex modulus



Even part of ReLU
Collapses complex phases

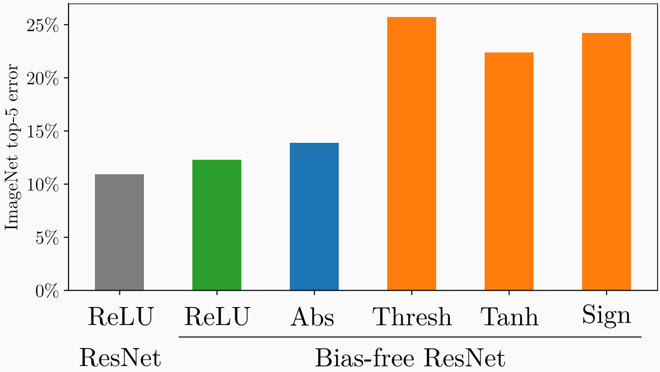


Concentrates multiplicative variability
Separates class means

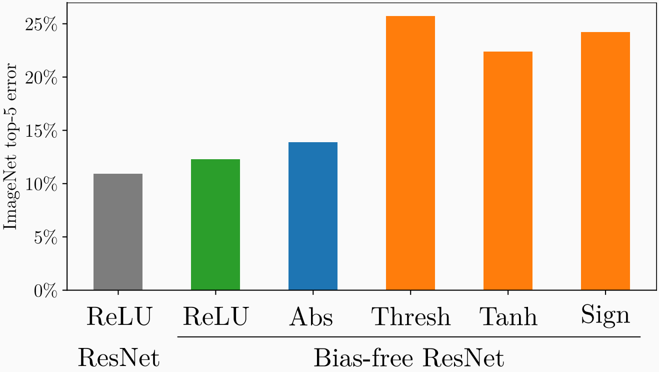


Computes support
Can be further sparsified

Phase collapse versus sparsity: numerical results



Phase collapse versus sparsity: numerical results



Phase collapse is sufficient to achieve good performance, while any non-linearity which preserves the phase is not. Phase collapse is thus also necessary.

How far can we further constrain the network?

Diagonalizing local translations

Known source of within-class variability: local translations

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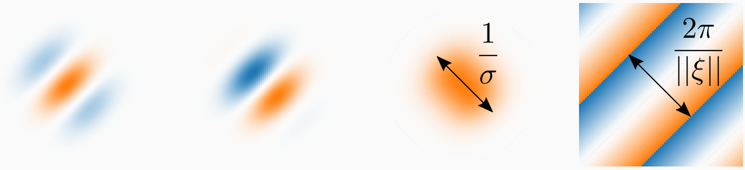


$\mathrm{Re}(\psi)$ $\mathrm{Im}(\psi)$ $|\psi|$ $\alpha(\psi)$

Small translations τ of an image x become phase shifts:

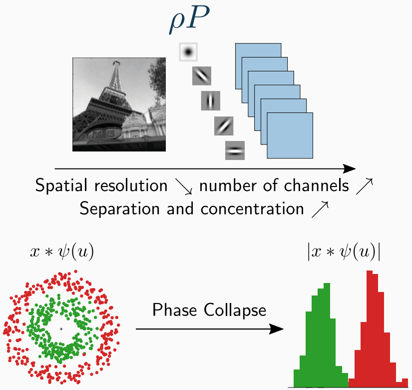
$$(\tau \cdot x) * \psi \approx e^{-i\xi\cdot\tau}(x * \psi)$$

with a relative error bounded by $\sigma|\tau|$: approximate diagonalization!

The phase collapse operator

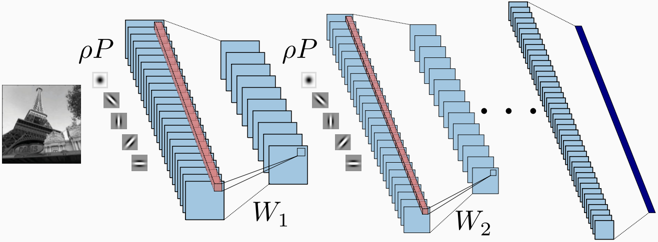
Constrain the spatial filters with the phase collapse operator:

$$\rho P x(u) = \Big(x * \phi(2u), \left(|x * \psi_\theta(2u)|\right)_\theta \Big)$$



- Mathematical definition: no learning
- Combines linear and non-linear invariants to local translations
- All the desired properties!
- What accuracy can we achieve with this?

Learned scattering network



- Simplified architecture with phase collapses and minimal learning
- No learned spatial filters nor biases
- Only one learned component: channel matrices at every layer
- Reaches ResNet-18 accuracy with only 11 layers

Zarka, G, and Mallat. Separation and concentration in deep networks. *ICLR*, 2021.
G, Zarka, and Mallat. Phase collapse in neural networks. *ICLR*, 2022.

Thank you!

- Paper: `arxiv.org/pdf/2110.05283.pdf`
- Code: `github.com/FlorentinGuth/PhaseCollapse`
- Email: `florentin.guth@ens.fr`

Paper

Code

Email